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**14. ABSTRACT** The rapid loss of radiation belt electrons in the main phase of geomagnetic storm is believed to be aided by EMIC waves, and is usually analyzed with quasi-linear theory. However, even moderate EMIC wave intensities ea\sily cause resonant electrons to respond nonlinearly, with drastically different results. We map out the region of nonlinear behavior with a single parameter, and show that both the direction and magnitude of scattering can be estimated by analytical expressions. The nonlinear interactions typically lead to advection toward large pitch angles, rather than diffusion toward the loss cone. This is expected to reduce the overall loss rate and greatly affect the distribution of trapped electrons.


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# Nonlinear interaction of radiation belt electrons with electromagnetic ion cyclotron waves

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[1] The rapid loss of radiation belt electrons in the main phase of geomagnetic storms is believed to be aided by EMIC waves, and is usually analyzed with quasi-linear theory. However, even moderate EMIC wave intensities easily cause resonant electrons to respond nonlinearly, with drastically different results. We map out the region of nonlinear behavior with a single parameter, and show that both the direction and magnitude of scattering can be estimated by analytical expressions. The nonlinear interactions typically lead to advection toward large pitch angles, rather than diffusion toward the loss cone. This is expected to reduce the overall loss rate and greatly affect the distribution of trapped electrons. **Citation:** Albert, J. M., and J. Bortnik (2009), Nonlinear interaction of radiation belt electrons with electromagnetic ion cyclotron waves, *Geophys. Res. Lett.*, *36*, L12110, doi:10.1029/2009GL038904.

## 1. Introduction

[2] Strong electromagnetic ion cyclotron (EMIC) waves, with typical intensities of $\sim 1-10$ nT, are commonly present in the outer radiation belts [*Meredith et al.*, 2003; *Fraser et al.*, 2006]. They are excited by unstable distributions of ring current ions, and can scatter both ring current and radiation belt ions into the loss cone. They may also strongly scatter radiation belt electrons, especially near the plasmapause and in high density plumes [*Thorne et al.*, 2005, 2006; *Shprits et al.*, 2008]. Such scattering has mostly been modeled as quasi-linear diffusion [*Lyons*, 1974; *Summers and Thorne*, 2003; *Gamayunov and Khazanov*, 2007; *Jordanova et al.*, 1996, 2008; *Miyoshi et al.*, 2008], which indicates that EMIC waves with magnetic field amplitudes of 1 nT, in conjunction with whistler mode hiss and chorus, give decay time scales for 1 MeV electrons of less than one day [*Albert*, 2003; *Li et al.*, 2007]. Similar calculations with stronger EMIC waves can reach the strong diffusion limit, where particles scatter into the loss cone faster than they can be removed by the atmosphere [*Shprits et al.*, 2009]. However, such strong amplitudes raise doubts about the validity of the quasi-linear framework, and a nonlinear treatment seems called for, as noted by *Millan and Thorne* [2007]. Such analysis for test particles has previously been developed and applied to large amplitude whistler mode chorus [e.g., *Nunn*, 1974; *Albert*, 2002; *Omura et al.*, 2008; *Bortnik et al.*, 2008]. Here, we use a similar approach to demonstrate the potentially nonlinear character of electron interactions with EMIC waves.

## 2. Inhomogeneity Parameter

[3] For field-aligned EMIC waves, the cyclotron resonance condition for electrons is

$$\omega - kv_{\parallel} = -\Omega_e/\gamma \tag{1}$$

where $\Omega_e = eB/mc$, $m$ is the electron rest mass, and $e$ is the electron charge (in absolute value). Resonances involving multiples of the cyclotron frequency do not play a role for field-aligned waves. Assuming the wave is generated near the equator and propagates away from it [e.g., *Fraser et al.*, 1996], the electron must be "co-streaming," i.e., directed toward high latitude. (This is opposite to the usual requirement for resonance with field-aligned whistler mode waves.)

[4] The dynamics of resonant wave-particle interactions have been treated by many authors [e.g., *Nunn*, 1974; *Inan et al.*, 1978]. *Albert* [1993, 2000] derived a "1 $\frac{1}{2}$ dimensional" Hamiltonian for the normalized first adiabatic invariant $I = \omega(p_\perp/mc)^2/2\Omega_e$, the wave-particle phase $\xi$, and distance $s$ along a field line, of the form

$$K(I,\xi,s) = K_0(I,s) + K_1(I,s)\sin\xi, \tag{2}$$

where $K_0$ describes adiabatic motion along a field line and $K_1$ captures the effect of the resonant wave. With field-aligned wave propagation, they take the form

$$\begin{aligned} K_0 &= \frac{kc}{\omega}(I - I_0) + \sqrt{(I-I_0)^2 - 1 - 2\Omega_e I/\omega}, \\ K_1 &= \frac{eB_w/mc}{kc}\tan\alpha, \end{aligned} \tag{3}$$

where $\alpha$ is the local particle pitch angle and $I_0$ is a constant.

[5] For fixed $s$ the Hamiltonian is analogous to that of a plane pendulum, whose phase portrait features two classes of periodic trajectories, with a separatrix between them. The frequency of small amplitude oscillations is $(K_1\,\partial^2K_0/\partial I^2)^{1/2}$, and the island width is $[4K_1/(\partial^2K_0/\partial I^2)]^{1/2}$. *Albert* [1993] considered the timescales for particle motion within a frozen phase portrait and for motion of the island itself, finding that the particle behavior was determined by their ratio,

$$R = \frac{\partial^2K_0/\partial I^2}{K_1(\partial^2K_0/\partial s\partial I)}. \tag{4}$$

For $|R| \gg 1$, "linear" motion is expected, with an effectively random value of the phase at resonance, leading

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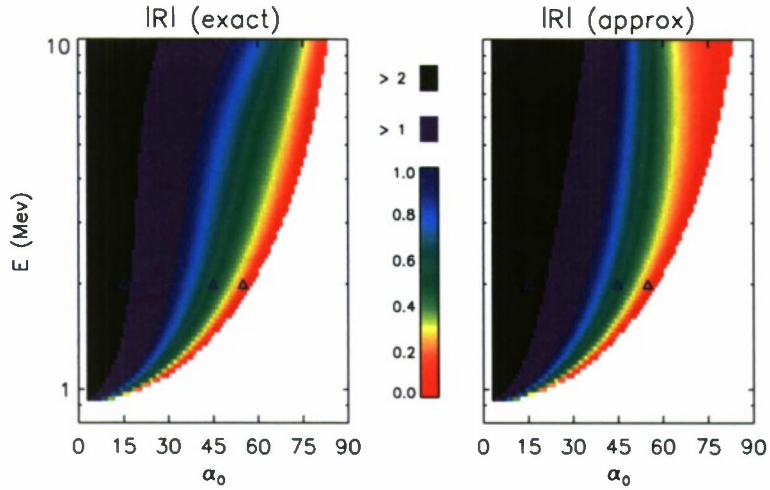


**Figure 1.** Inhomogeneity parameter $R$ versus equatorial pitch angle and energy, for a model of 2 nT EMIC waves in a high density plume at $L = 4$. The full equation (5) was used for the left-hand plot, and the approximate equation (7) was used on the right. The triangles indicate the particles simulated numerically. Nonlinear interactions are expected for $R < 1$.

to diffusion in $I$, while $|R| \ll 1$ leads to essentially nonlinear motion, with phase bunching (with or without phase trapping), and deterministic changes in $I$. $R$ represents a competition between the strength of the wave, in $K_1$, and the inhomogeneity of the background magnetic field (and plasma properties), which enters through the derivative with respect to $s$ (effectively, latitude $\lambda$).

[6] *Omura et al.* [2008] recently presented a related analysis for field-aligned whistler-mode chorus waves, working directly with the resonance-averaged equations of motion, and obtained a related nonlinear oscillator equation with the same properties. Repeating their derivation for arbitrary (but field-aligned) waves yields an "inhomogeneity parameter" which coincides exactly with $R$, and can be written explicitly as

$$R = \frac{-B}{B_w}\frac{\mu}{\mu^2-1}\frac{c}{v_\perp}\frac{c}{\omega}\left\{\left[\mu\frac{\gamma}{2}\frac{\omega}{\Omega_e}\frac{v_\perp^2}{c^2} - \frac{v_\parallel}{c}\right]\frac{1}{\Omega_e}\frac{\partial\Omega_e}{\partial s} - \mu\frac{\gamma}{2}\frac{\omega}{\Omega_e}\frac{v_\parallel^2}{c^2}\frac{1}{\mu^2}\frac{\partial(\mu^2)}{\partial s}\right\}, \quad (5)$$

evaluated at resonance. Here $\mu$ is the refractive index, $kc/\omega$.

[7] For parallel-propagating EMIC waves, $\mu^2$ is given by the Stix L parameter [*Stix*, 1962], which is conveniently written as

$$\mu^2 = 1 - \frac{\omega_{pe}^2}{\omega^2}\left\{\frac{\omega}{\omega+\Omega_e} + \sum_i \frac{\eta_i Z_i^2}{\beta_i}\frac{1}{M}\frac{\omega}{\omega-\Omega_i}\right\}, \quad (6)$$

where $M$ is the electron-to-proton mass ratio $m/m_p$, $i$ indexes the ion species, $\beta_i = m_i/m_p$, $\eta_i = n_i/n_e$, and $Z_i$ is the charge of ion $i$. Quasineutrality requires $\sum_i Z_i\eta_i = 1$. EMIC waves only propagate where $\mu^2 > 0$, which occurs in frequency bands bounded below by cutoffs and above by resonances at the ion cyclotron frequencies.

[8] We take the ions to be $H^+$, $He^+$, and $O^+$, so that $\beta_H = 1$, $\beta_{He} = 4$, $\beta_O = 16$, with all $Z_i = 1$. Typical storm-time values of the ion concentrations are $\eta_H = 0.77$, $\eta_{He} = 0.20$, $\eta_O = 0.03$ [*Jordanova et al.*, 2008]. The value of $\mu$ in equation (6) will be dominated by the term corresponding to the appropriate ion band. Making this approximation, using a small-latitude approximation of the dipole magnetic field, and ignoring density variation in equation (5) yield the estimate

$$R \approx \frac{-9\lambda}{2L}\frac{mc}{eB_w}\frac{c}{R_e}\frac{p}{mc}\frac{\cos^2\alpha}{\sin\alpha}\frac{1}{1-\omega/\Omega_i}. \quad (7)$$

From now on, it will be understood that "$R$" refers to $|R|$.

[9] Figure 1 shows $R$ as a function of energy and equatorial pitch angle $\alpha_0$ for field-aligned, 2 nT EMIC waves in the helium band, evaluated in a high density plume at $L = 4$, with $\omega_{pe}/\Omega_e = 15$. The frequency $\omega/2\pi$ is taken to be about 1.84 Hz, so that $\omega/\Omega_i = 0.96$ at the equator. Such a large ratio is necessary to bring the minimum resonant energy down to about 1 MeV [*Summers and Thorne*, 2003; *Albert*, 2003]. Equations (5) and (7) give very similar results for $R$. Near the loss cone, the interactions are expected to be linear (that is, diffusive, as discussed below). However, for a wide range of pitch angle and energy values, the interactions are expected to be nonlinear, so that quasi-linear diffusion will not be a valid description.

## 3. Equations of Motion and Analytical Estimates

[10] To test these ideas, we follow particles numerically, using the gyroresonance-averaged equations of motion following *Chang and Inan* [1983]. These equations take the form

$$\begin{aligned}
\frac{dp_\parallel}{dt} &= \frac{eB_w}{c}\frac{p_\perp}{m\gamma}\sin\phi - \frac{p_\perp^2}{2m\gamma B}\frac{\partial B}{\partial s}\\
\frac{dp_\perp}{dt} &= \frac{eB_w}{c}\left(\frac{\omega}{k} - \frac{p_\parallel}{m\gamma}\right)\sin\phi + \frac{p_\perp p_\parallel}{2m\gamma B}\frac{\partial B}{\partial s}\\
\frac{d\phi}{dt} &= \left(\frac{kp_\parallel}{m\gamma} - \omega - \frac{\Omega_e}{\gamma}\right) + \frac{eB_w}{c}\left(\frac{\omega}{k} - \frac{p_\parallel}{m\gamma}\right)\frac{\cos\phi}{p_\perp}\\
\frac{ds}{dt} &= \frac{p_\parallel}{m\gamma}
\end{aligned} \quad (8)$$

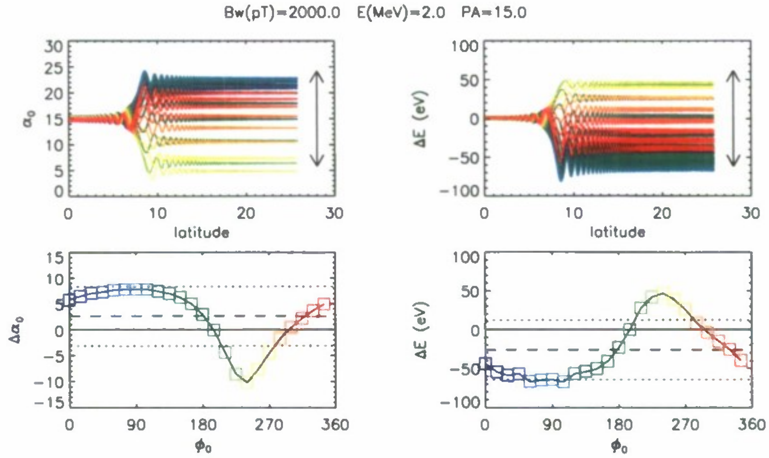


**Figure 2.** Trajectories of 2 MeV electrons with $\alpha_0 = 15°$, interacting with 2 nT EMIC waves in a high density plume. Even for these large amplitude waves, the interaction is fairly linear, leading to nearly symmetric scattering in $\alpha_0$ and in $E$.

In a dipole field, $s$ is easily transformed to latitude $\lambda$. Both the wave frequency $\omega$ and magnetic field strength $B_w$, as well as the background plasma density, are treated as constant. The particles were not allowed to reach the location of the crossover frequency, where the Stix parameter $D = 0$, and the wave can switch from left hand to right hand circular polarization [*Stix*, 1962]. The wave-particle phase $\phi$ differs from $\xi$ in equation (2) only by sign and phase conventions.

[11] The Hamiltonian analysis of *Albert* [1993] yields analytical estimates of the net, nonadiabatic change in $I$ due to either linear or nonlinear resonant interactions. These are:

$$\Delta I_{lin} = -K_1\sqrt{\frac{2\pi}{|\partial^2 K_0/\partial s\partial I|}}\cos\tilde{\xi}, \quad R > 1,$$
$$\Delta I_{NL} = \frac{8}{\pi}\sqrt{\left|\frac{K_1}{\partial^2 K_0/\partial I^2}\right|}, \quad R < 1. \tag{9}$$

Here $\tilde{\xi}$ is related to the value of $\xi$ evaluated right at resonance, and is effectively random. Thus in the linear regime, $I$ suffers a random walk with zero mean, or diffuses, while in the nonlinear regime the resonant value of $\xi$ is well defined and $I$ experiences systematic drift, or advection. Corresponding changes in energy and $\alpha_0$ are [*Albert*, 2000]

$$\Delta\alpha_0 = \frac{\gamma\sin^2\alpha_0 + \Omega_{eq}/\omega}{(p/mc)^2\sin\alpha_0\cos\alpha_0}\Delta I, \quad \frac{\Delta E}{mc^2} = -\Delta I, \tag{10}$$

where $\Omega_{eq}$ is the equatorial value of $\Omega_e$. Since $\omega \ll \Omega_e$, pitch angle changes will be much larger than normalized energy changes.

## 4. Results

[12] First, the trajectories of 24 particles interacting with a small amplitude wave, $B_w$ = 5 pT, were calculated according to equation (8). The particles were started at the equator with $E$ = 2 MeV, $\alpha_0 = 15°$, and a uniform spread in initial phase, $\phi_0$. For this arrangement, $R$ at the expected resonance location is nearly 1000. After passing through the resonance, both $\alpha_0$ and $E$ are sharply scattered. The distributions of $\Delta\alpha_0$ and $\Delta E$, though not shown, are symmetric about zero and depend sinusoidally on the initial phases, which should be similar to the phases just before the resonant interactions. Furthermore, the scattering amplitudes agree quite well with the prediction $\Delta I_{lin}$.

[13] Figure 2 shows a similar simulation with $B_w$ = 2 nT, which gives $R$ = 2.4. This distribution of changes is also fairly sinusoidal, though not perfectly so, and the amplitudes of the changes are well predicted by $\Delta I_{lin}$, shown as the double-arrowed line. The maximum change in $\Delta\alpha_0$ is about 10°, which is nearly enough to move the particles into the loss cone in one bounce. The pitch angle diffusion coefficient, $\langle(\Delta\alpha_0)^2\rangle/\tau_b \approx 3 \times 10^3$ day$^{-1}$, is in reasonable agreement with calculations based on broadband quasi-linear theory applied to similar models of EMIC waves at $L$ = 4.5 [*Li et al.*, 2007; *Albert*, 2008]. The bottom two panels show the expected, nearly sinsusoidal dependence on initial phase, as well as mean changes (dashed lines) and mean changes plus or minus one standard deviation (dotted lines).

[14] In Figure 3, the initial pitch angle of the 2 MeV particles is changed to $\alpha_0 = 45°$, for which $R$ = 0.54, a moderately nonlinear value. In fact, the distribution of changes is very different from above, with most particles tracing very similar paths. This is due to the underlying "bunching" of wave-particle phases as the particles encounter and circumnavigate the separatrix between trapped and untrapped trajectories (see, e.g., *Nunn* [1974], *Albert* [1993, 2000], *Omura et al.* [2008] for details). Some paths are exceptions, reflecting the fact that $R$ is not very much less than 1. Nevertheless, the predicted changes based on $\Delta I_{NL}$, indicated by the plus symbols, give good estimates of the maximum, and most common, values. Repeated interactions would cause the particle distributions to drift, not diffuse, in pitch angle and energy; the cumulative changes would grow proportionally to time, not as $t^{1/2}$. Also note that the typical change in $\alpha_0$ is positive, away from the loss cone. This is expected to cause lower phase space density at the relevant pitch angles, and lower overall loss rates, than estimates based on pure diffusion.

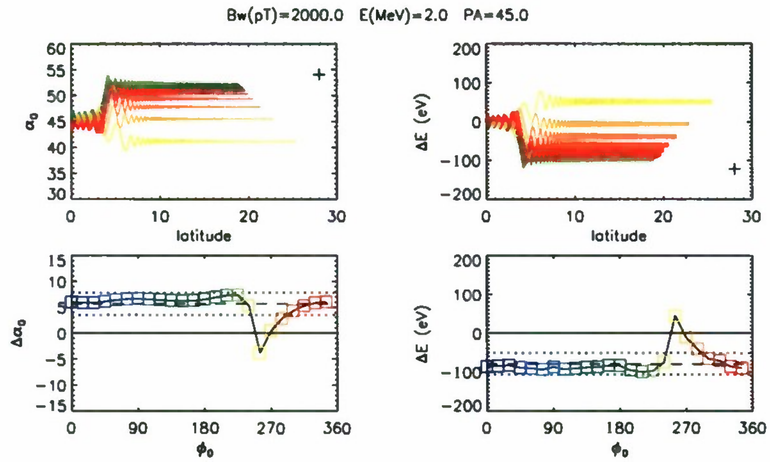


**Figure 3.** Trajectories of 2 MeV electrons with $\alpha_0 = 45°$, interacting with 2 nT EMIC waves in a high density plume. For these particles, the interaction is nonlinear, leading to phase bunching with systematic increases in $\alpha_0$ and decreases in $E$.

[15] None of the particles in Figure 3 exhibit phase trapping, whereby particles cross the separatrix and spend many phase oscillations within it. This is not surprising since the particles are moving away from the equator, as required for resonance with the EMIC waves, and so experience increasing magnetic field inhomogeneity. Even if $R < 1$ at resonance, $R$ will be increasing, which is not conducive to trapping [*Albert*, 2000]. On the other hand, particles may start out inside the separatrix. Figure 4 shows 2 MeV particles starting with $\alpha_0 = 55°$ at the equator. The linear resonance condition, equation (1), is satisfied at $\lambda = 1.7°$, where $R = 0.2$. About half the particles behave nonlinearly in accordance with $\Delta I_{NL}$, as shown by the plus marks. However, almost half the particles experience even larger, and oppositely signed changes, which develop over a sustained interval of nonadiabatic motion, as is characteristic of phase trapping. Evidently the initial conditions are near enough to resonance that some particles complete several phase oscillations before becoming nonresonant. Detailed examination of the trajectories verifies that for the trapped particles, $\phi$ oscillates in a limited range starting at $t = 0$. Physically, this would occur if $B_w$, which has been held constant, grew rapidly to "envelop" regions of phase space containing nearly resonant particles.

## 5. Summary

[16] We have demonstrated that the cyclotron resonant interaction of relativistic electrons with field-aligned EMIC waves can be highly nonlinear. Regions of expected nonlinear behavior were mapped out with the parameter $R$, using a realistic magnetic field amplitude of 2 nT. Moreover, we have shown that previously developed theoretical expressions for the expected pitch angle and energy changes give good estimates of the behavior seen in numerical simulation of the particle motion. Phase bunching without trapping, the more commonly expected version of nonlinear behavior, leads to rapid pitch angle increases, away from the loss cone, with small changes in energy. In combination with diffusion, this could lead to major, complex effects on

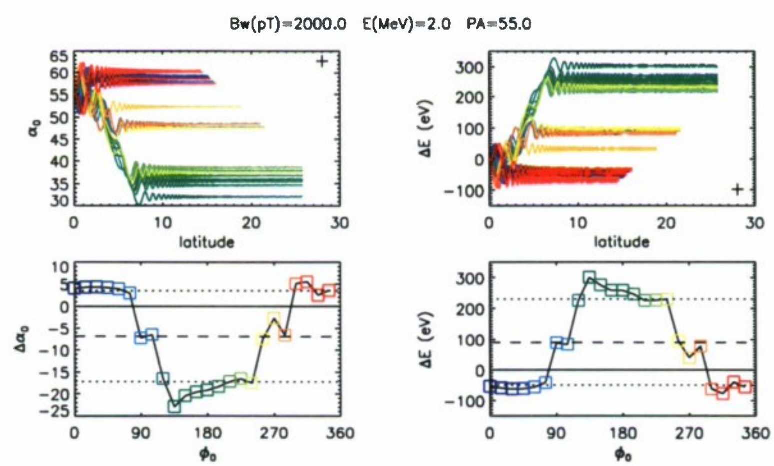


**Figure 4.** Trajectories of 2 MeV electrons with $\alpha_0 = 55°$, interacting with 2 nT EMIC waves in a high density plume. For these particles, the interaction is nonlinear and some particles are initially phase trapped, giving both systematic increases and sustained decreases in $\alpha_0$, and oppositely directed changes in $E$.

both the loss rate and the trapped particle distribution. Since EMIC waves are believed to play a major role in the loss of radiation belt electrons during storms, such behavior should be accounted for in future models. One approach would be to use the analytical expressions in a combined diffusion-advection formulation.

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